Fast Subgrid FD-TD Matrix Pencil Technique for the Rigorous Analysis of Resonant 3D Microwave Structures

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Abstract— **An improved FD-TD technique is described for the rigorous analysis of highly resonant 3D microwave structures, such as dielectric resonator filters. The method is based on a recursive subgrid procedure, utilizes a robust orthogonalization technique, and employs the matrix pencil algorithm for the frequency-transformation. The S-parameters are extracted from least-square-solutions of over-determined systems of equations for each port. The efficiency and accuracy of the presented FD-TD technique is demonstrated at the example of LANGER's dielectric resonator reference filter. The analysis of modified filters with up to four dielectric resonators illustrates the flexibility of the method.**

I. Introduction

DUE to its simplicity and flexibility, the finite difference time domain (FD-TD) method is well established for solving a wide variety of electromagnetic problems [1] - [10]. As usual microwave components, such as filters, often include geometries of very different shape, the numerical effort for accurate results can be high if a uniform mesh is used. Moreover, resonant structures, in particular, require a large number of time steps. For rigorous microwave component simulations with the FD-TD method, therefore, adequate and stable subgrid techniques combined with efficient modal S-parameter extraction methods are highly desirable.

Usual graded mesh techniques [9], [10], often lead to unnecessarily fine mesh discretizations in homogeneous areas of low field gradients, due to the topology of the grid. Moreover, the stable maximum time step depends on the smallest cell used, and still rather high memory and computation time requirements are usually necessary.

Locally refined meshes have been presented in [2], [3], [4], [5]. The reported subgrid FD-TD algorithms, however, require additional interpolation schemes at the grid-interfaces which often reduces the flexibility by prohibiting arbitrary combinations of different sized cells. Moreover, depending on the type of interpolation, this can violate the divergence relations, which then results in unstable formulations.

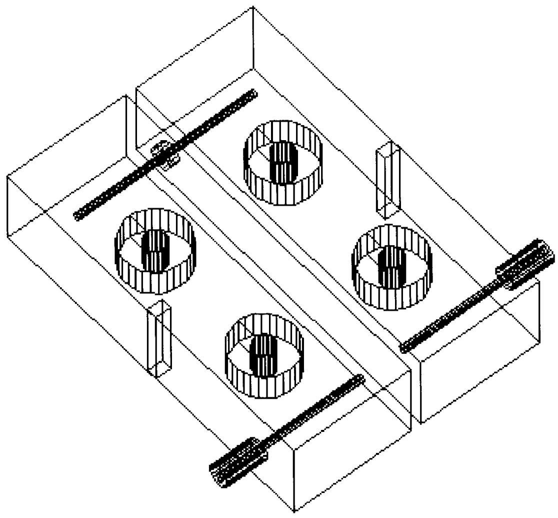


Fig. 1. Dielectric resonator filter example investigated with the presented subgrid FD-TD matrix pencil technique: Four-resonator planar filter with additional coaxial resonators and irises.

This paper presents an advanced combination of the generalized subgrid-technique, recently introduced by the authors in [11], with the matrix-pencil method [12] and with a new approach for the S-parameter extraction. Moreover, compared to earlier applications of the matrix-pencil technique to the FD-TD method [8], in this paper we use a different value for the so called "pencil-parameter" which increases the convergence of the frequency-transformation. Furthermore, improved least-square formulations are used for the purpose of separating the propagated and reflected waves at the terminal ports of the structure under investigation.

Compared to the traditional approach using signals at only 2 different waveguide cross-sections, the over-determined least-squares problem (using typically 5 input-signals from different cross-sections) yields naturally more robust results. These two enhancements result in a further reduction of the total number of time-steps for the FD-TD simulation, and, hence, make the subgrid FD-TD matrix-pencil technique a versatile, fast and reliable simulation tool for practical filter design applications.

The efficiency of the presented method is demonstrated by a low CPU time for the accurate analysis of LANGER's two resonator reference filter [14]. The simulation of several modified filters including irises, additional coaxial resonators and up to four resonators (Fig. 1) shows the flexibility of the method. Its accuracy is verified by comparison with available measurements.

II. THEORY

Grid generation – Based on a progressive 2 : 1 cell ratio for the subgrids [11], we use a recursive grid-generation procedure. A given object is first subdivided into cells of a largest level l such that all corners of the cell are still inside the object. To discretize the space with the intended finer mesh, bounded by the object and the level l cells, the same procedure is repeated using cells of the next finer level $l+1$, until the maximum specified level l_m is obtained.

From the allocated cells, two grids are derived, which are referred to as the main and the dual grid. The main grid is defined by the corners, and the dual grid is defined by the centers of the allocated cells [11]. The dual mesh involves quadrangular and triangular cells. The main grid is directly orthogonalized against the dual grid (Fig. 2). In the three dimensional (3D) case, the orthogonalization of the main and the dual grid is performed analogously.

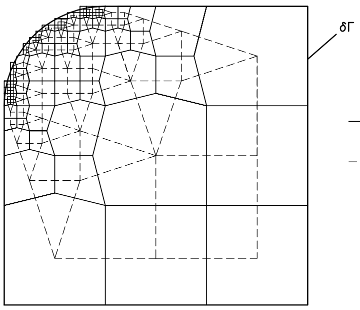


Fig. 2. 2D subgrid generation of a FD-TD domain.

For a discretization containing cells of the maximal level l_m, a total number of $\sum_{l=1}^{l_m+1} 2^l$ different e and h updated computations have to be performed to proceed one step ${}^0\Delta t = \Delta t 2^{l_m}$ in time in the coarsest grid (of level 0). This update sequence is repeated after 2^{l_m} time steps.

In contrast to the spatial and/or temporal interpolation techniques hitherto reported, the generalized subgrid FD-TD method presented, satisfies implicitly for each cell – in both the main and the dual grid – the divergence relation. Analogous to the standard non subgrid FD-TD formulation, the integral contribution of each vertex of the cell appears twice but with different signs so that the sum is zero. This yields for all investigated cases stable results as has been tested even at highly resonant structures, e.g. dielectric resonator filters, with up to several millions of time iterations.

The time step in each grid level l is limited by the Courant-Friedrichs-Lewy (CFL) condition [13] for the nonorthogonal FD-TD method [6].

Matrix pencil technique – The powerful combination of the FD-TD method and the matrix pencil technique [12], [8] is used for the full-wave analysis of waveguide structures of more general shape. This technique avoids the drawbacks of the slow convergence of the standard FFT formulation, and of the very small time increments, when applied to structures which are small compared with the wavelength.

With the data vectors $\boldsymbol{x_t}$ of length $N-L$ for the noiseless signal x_k

$$\boldsymbol{x}_t = [x_t, x_{t+1}, \ldots, x_{N-L+t-1}]^T \ . \tag{1}$$

the matrices X_0 and X_1 are defined

$$\underset{(N-L)\times L}{X_0} = [\boldsymbol{x}_{L-1}, \boldsymbol{x}_{L-2}, \ldots, \boldsymbol{x}_0] \tag{2}$$

$$\underset{(N-L)\times L}{X_1} = [\boldsymbol{x}_{L}, \boldsymbol{x}_{L-1}, \ldots, \boldsymbol{x}_1] \, . \tag{3}$$

Every pole $z_t, t = 1, \ldots, M$ reduces the rank of the 'matrix-pencil' $X_1 - z_t X_0$ exactly by 1 [12], if the 'pencil-parameter' L is chosen to be $M \leq L \leq N-M$; in this case, the matrix-pencil $X_1 - z_t X_0$ is of rank $M-1$. Otherwise, the rank of the matrix-pencils remains M. Each pole is an eigenvalue of the generalized eigenvalue problem

$$(X_1 - z_t X_0)\, \boldsymbol{q}_t = 0 \tag{4}$$

with non quadratic matrices. This equation is transformed into a standard eigenvalue problem of a quadratic matrix by multiplication from left with the pseudo inverse [12] X_0^+ of the matrix X_0:

$$\left(X_0^+ X_1 - z_t I\right) \boldsymbol{q_t} = 0, \tag{5}$$

which does not change the eigenvalues and -vectors. Several applications of the matrix-pencil technique to FD-TD time-signals have shown, that an optimal choice for the pencil parameter L is about 0.46…0.48 times the total time-steps applied. This leads to a faster convergence of the spectra in terms of the total time-steps involved.

The S-parameters are computed from the propagated and reflected wave-amplitudes a_i, b_i at the terminal-ports of the structure under investigation. Typically, the wave-amplitudes are obtained by separating the total field w_i at two different cross-sections.

In the present paper, a larger number (typically five) of total-field values w_i are taken into account. The resulting, over-determined system of equations is solved by means of least squares. This more robust approach is required due to not negligible approximation errors in the spectra computed by the matrix-pencil method.

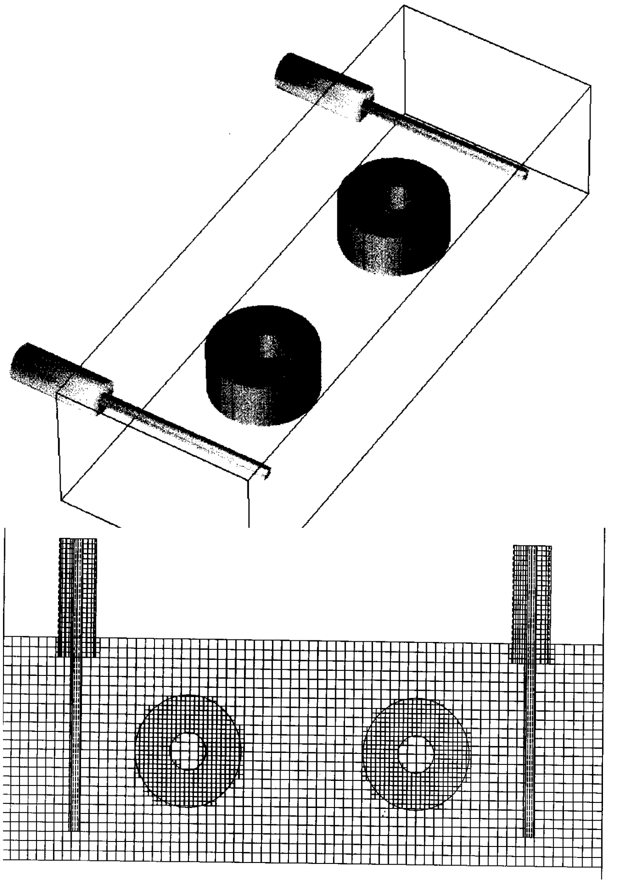


Fig. 3. LANGER's dielectric resonator filter. (a) Filter structure [14]. (b) Subgrid mesh.

III. Results

The described 3D FD-TD subgrid matrix pencil technique has been tested first at LANGER's dielectric resonator reference filter [14], cf. Fig. 3a. The subgrid discretization is illustrated in Fig. 3b.

Figs. 4 shows the computed S-parameters (Fig. 4a) compared to measurements (Fig. 4b) reported in [14]. Good agreement may be stated. The CPU time for the total simulation using the high resolution of 4000 frequency sample points (Fig. 4a) was only about 40 min on a SGI Origin 200 workstation. The spikes at about 5.8 and 6.6 GHz (Fig. 4b) are clearly reproduced.

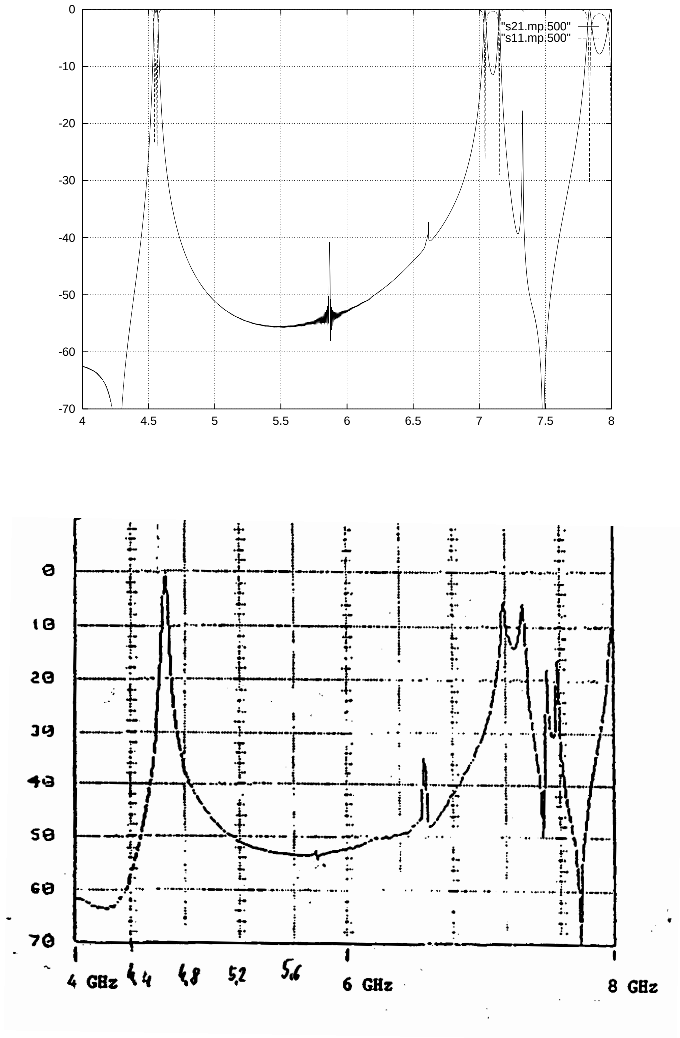
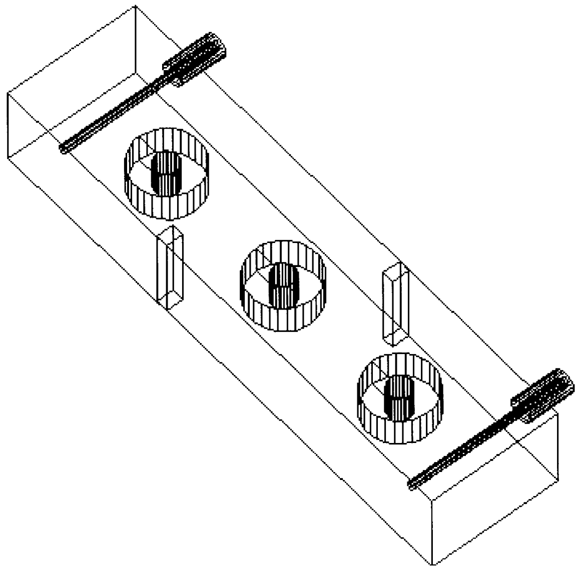


Fig. 4. LANGER's dielectric resonator filter. (a) Computed S-parameters compared to measurements (b) presented in [14].

Fig. 5a shows a planar dielectric resonator filter with three resonators coupled by irises. In Fig. 5b, the calculated scattering parameters are presented. Moreover, the transmission coefficient S_{21} (solid line) is compared with S_{21} of the same filter without irises (dashed line) illustrating the improvements concerning the spurious behavior by the additional irises.

An example for demonstrating the flexibility of the presented method is a four-resonator filter (Fig. 1) coupled by irises and additional coaxial resonators. The computed transmission coefficient S_{21} (solid line) is compared in Fig. 6 with S_{21} of the filter without irises (dashed line) and slightly longer coaxial resonators. This again makes the improvements evident concerning the spurious behavior by additional irises and, here, coaxial line coupling adjustments.



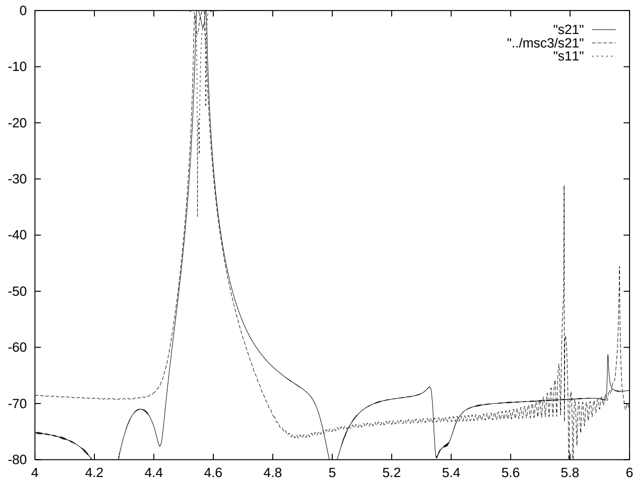


Fig. 5. Planar three resonator filter (dielectric material $\epsilon_r = 38$). (a) Structure. (b) Calculated S-parameters with (solid line) and without (dashed line) additional irises.

IV. Conclusion

An improved, fast and stable FD-TD subgrid matrix pencil technique is described for the rigorous analysis of highly resonant 3D microwave structures, such as dielectric resonator filters. The technique yields low CPU times and improved accuracy.

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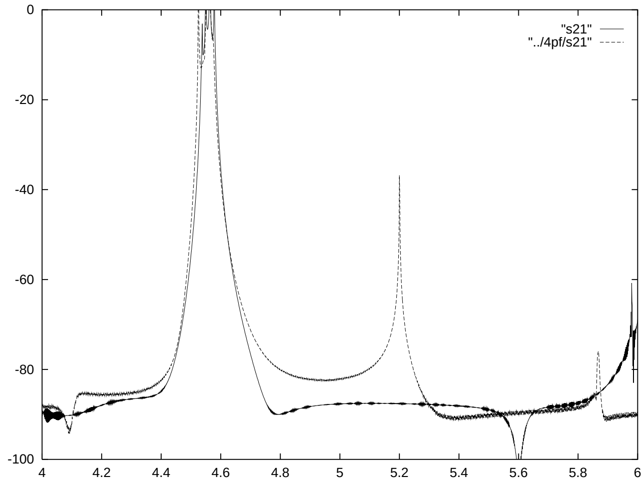

Fig. 6. Planar four resonator filter (dielectric material $\epsilon_r = 38$) of Fig. 1: Calculated S21-parameters with (solid line) and without (dashed line) additional irises.

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